

## Scheme of Work

# Cambridge International A Level Further Mathematics

## 9231 Paper 2

For examination from 2017–2019



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## Introduction

This scheme of work has been designed to support you in your teaching and lesson planning. It is important to have a scheme of work in place in order for you to ensure that the syllabus is covered fully and this scheme can be adapted to suit your institution and the levels of ability and learning preferences of your learners.

Timings for activities and feedback are left to the judgement of the teacher, according to the level of the learners and size of the class. The length of time allocated to a task forms another possible area for differentiation.

The nature of this paper is that it consists of a Mechanics and Statistics section. It is entirely appropriate to study either Mechanics or Statistics first, or to teach the material in parallel if that fits with the Centre's teaching arrangements.

### Guided learning hours

Guided learning hours give an indication of the amount of contact time teachers need to have with learners to deliver a particular course. Our syllabuses are designed around 360 hours for Cambridge International A Level. The number of hours may vary depending on local practice and your learners' previous experience of the subject. The table below give some guidance about how many hours are recommended for each topic.

Topic (Syllabus ref)	Suggested teaching time	Suggested teaching order
1. Momentum and impulse	It is recommended that this unit should take about 15 hours	<b>M1</b>
2. Circular motion	It is recommended that this unit should take about 20 hours	<b>M3</b>
3. Equilibrium of a rigid body under coplanar forces	It is recommended that this unit should take about 20 hours	<b>M2</b>
4. Rotation of a rigid body	It is recommended that this unit should take about 25 hours	<b>M5</b>
5. Simple harmonic motion	It is recommended that this unit should take about 20 hours	<b>M4</b>
6. Further work on distributions	It is recommended that this unit should take about 8 hours	<b>S2</b>
7. Inference using normal and t-distributions	It is recommended that this unit should take about 25 hours	<b>S3</b>
8. $\chi^2$ -tests	It is recommended that this unit should take about 15 hours	<b>S4</b>
9. Bivariate data	It is recommended that this unit should take about 12 hours	<b>S1</b>

### Resources

The resource list for this syllabus is listed at [www.cie.org.uk](http://www.cie.org.uk). There are no endorsed textbooks for this syllabus at present, but the list includes useful textbooks which cover much of the syllabus as well as other references.

### Teacher support

Teacher Support <https://teachers.cie.org.uk> is a secure online resource bank and community forum for Cambridge teachers, where you can download specimen and past question papers, mark schemes and other resources. We also offer online and face-to-face training; details of forthcoming training opportunities are posted online. This scheme of work is available as PDF and an editable version in Microsoft Word format; both are available on Teacher Support at <https://teachers.cie.org.uk>. If you are unable to use Microsoft Word you can download Open Office free of charge from [www.openoffice.org](http://www.openoffice.org).

### Websites

This scheme of work includes website links providing direct access to internet resources. Cambridge International Examinations is not responsible for the accuracy or content of information contained in these sites. The inclusion of a link to an external website should not be understood to be an endorsement of that website or the site's owners (or their products/services).

Teachers can register free for the STEM website but subsequent to the production of this Scheme of Work the Integral website is now only available via paid subscription.

The website pages referenced in this scheme of work were selected when the scheme of work was produced. Other aspects of the sites were not checked and only the particular resources are recommended.

**How to get the most out of this scheme of work – integrating syllabus content, skills and teaching strategies**

We have written this scheme of work for the Cambridge International A Level Further Maths 9231 Paper 2 syllabus to provide ideas and suggestions of how to cover the content of the syllabus. Coverage is not always complete and some topics are included which are not required for the Cambridge syllabus. References to other syllabuses are given in this document but these are mainly Mechanics and Statistics components from OCR. We have designed the following features to help guide you through your course.

**Learning objectives** help your learners by making it clear the knowledge they are trying to build. Pass these on to your learners by expressing them as 'We are learning to / about...'

**Suggested teaching activities** give you lots of ideas about how you can present learners with new information without teacher talk or videos. Try more active methods which get your learners motivated and practising new skills.

Syllabus ref	Learning objectives	Suggested teaching activities
1. Polynomials and rational functions	Recall and use the relations between the roots and coefficients of polynomial equations, for equations of degree 2, 3, 4 only	<p><b>Prior knowledge:</b> Learners will be able to solve quadratic equations by factorising to identify integer roots</p> <p><b>Main theme:</b> Learners can investigate the connection between the roots of a quadratic equation and the coefficients, and then justify their findings by multiplying out the linear factors and matching coefficients. It's easier to start with the coefficient of <math>x^2</math> equal to 1. This can then be extended to other coefficients, and to cubics and quartics. The simple identity <math>\sum \alpha^2 \equiv (\sum \alpha)^2 - 2 \sum \alpha\beta</math> is worth teaching, but the more complicated formulae are sometimes misquoted, so it's probably better to use the equations themselves for higher powers. The Integral website <a href="http://integralmaths.org">http://integralmaths.org</a> has some notes and examples for polynomials up to degree 3. Look at MEI FP1 which goes up to quartics. or OCR Further Pure 1 which could be used for independent study (I) or revision up to cubics (F). There are also notes, with short exercises (F), on this topic on the Community Resources area of Teacher Support <a href="https://teachers.cie.org.uk">https://teachers.cie.org.uk</a></p> <p><b>Extension activity:</b> Learners can investigate and find a general rule.</p>

**Extension activities** provide your more able learners with further challenge beyond the basic content of the course. Innovation and independent learning are the basis of these activities.

**Independent study (I)** gives your learners the opportunity to develop their own ideas and understanding with direct input from you.

**Past examination papers, Specimen Papers and Mark Schemes** are available for you to download at

<https://teachers.cie.org.uk>

Using these resources with your learners allows you to check their progress and give them confidence and understanding.

**Formative assessment (F)** is on-going assessment which informs you about the progress of your learners. Don't forget to leave time to review what your learner has learnt, you could try question and answer, tests, quizzes, 'mind maps', or 'concept maps'. These kinds of activities can be found in the scheme of work.

## Momentum and impulse

Syllabus ref	Learning objectives	Suggested teaching activities
1. Momentum and impulse	<p>Recall and use the definition of linear momentum and show understanding of its vector nature;</p> <p>Recall Newton's experimental law;</p> <p>Use conservation of linear momentum and/or Newton's experimental law to solve problems;</p> <p>Use the definition of the impulse...(in one dimension only).</p>	<p><b>Prior knowledge:</b> Learners should already be familiar with the Kinetic energy of a system of particles.</p> <p><b>Introduction:</b> For a simple introduction to the principles in this topic, there is a vast array of material to allow learners to become familiar with basic concepts such as:  <a href="http://www.youtube.com/watch?v=WjHoYdvairw">www.youtube.com/watch?v=WjHoYdvairw</a>  <a href="http://www.youtube.com/watch?v=l6FYmoBB87w">www.youtube.com/watch?v=l6FYmoBB87w</a></p> <p><b>Main theme:</b> Learners should be familiar with the use of equations based on the conservation of linear momentum and Newton's experimental laws. They need to be familiar with the meaning of the terms 'perfectly elastic' and 'inelastic' when <math>e = 1</math> and <math>0</math> respectively. They should appreciate that <math>e=0</math> implies that the particles coalesce while <math>e = 1</math> results in no loss of kinetic energy.</p> <p>Problems will be restricted to the direct impact of two smooth spheres or the impact of a smooth sphere with a fixed surface. It is vital that learners appreciate the key importance of using a consistent and straightforward style of notation that they, and others, can follow easily. In problems involving a fixed surface, learners should be familiar with either using components of velocity for a single impact or considering the components of velocity in the course of more than one impact.</p> <p>Problems set may, as indicated, involve impulse or the loss of kinetic energy in one or more collisions.</p> <p>Detailed notes are available at <a href="http://integralmaths.org/">http://integralmaths.org/</a> along with a range of exercises and examples.  <a href="http://integralmaths.org/pluginfile.php/13656/mod_resource/content/0/ocrm2i1n.pdf">http://integralmaths.org/pluginfile.php/13656/mod_resource/content/0/ocrm2i1n.pdf</a>  Simple examples can be examined as an activity(I) using: <a href="http://www.geogebra.org/m/m1boNBIV">www.geogebra.org/m/m1boNBIV</a></p> <p>As with all work in this area of applied mathematics, the importance of consolidation and practice cannot be emphasised enough and a significant proportion of the available time must be devoted to attempting problems of varying difficulty.</p> <p><b>Extension activity:</b> There are a range of problems involving a variety of unusual contexts available in <a href="http://www.nrich.maths.org">www.nrich.maths.org</a> (E).</p>

**Past examination papers**

Past examination papers and mark schemes are available to download at <https://teachers.cie.org.uk>

Jun 14 Paper 21/22 Q1 Collision particles

Jun 14 Paper 23 Q1 Collision particles and impulse

Jun 15 Paper 21/22 Q5 Collision particles

Jun 15 Paper 23 Q1 Collision particles /Q2 Collision barrier

Nov 15 Paper 21 Q2 Collision barrier

Jun 16 Paper 21/22 Q2 Collision barrier and particles

Jun 16 Paper 23 Q5 Collision particles

## Circular motion

Syllabus ref	Learning objectives	Suggested teaching activities
2. Circular motion	<p>Recall and use the radial and transverse components of acceleration for a particle moving in a circle with variable speed;</p> <p>Solve problems which can be modelled by the motion of a particle in a vertical circle without loss of energy.</p>	<p><b>Prior knowledge:</b> Learners should be familiar with the constant angular acceleration formulae and the use of conservation of energy in simple systems. They should have a good understanding of motion in a horizontal circle which forms a basic introduction to this topic.</p> <p><b>Main theme:</b> The crucial point is that there is no transverse force. The only forces acting are weight, which always acts vertically downwards, and a force towards the centre of the circle, which keeps the object on its circular path. The resultant of these two forces has a component towards the centre and a transverse component. Learners should be able to solve problems where the information is given in terms of the linear or angular velocity components and be able to seamlessly switch between them. They should be aware of a variety of methods for the derivation of these results. Notes are available at <a href="http://integralmaths.org/pluginfile.php/15312/mod_resource/content/0/ocrm3c1n.pdf">http://integralmaths.org/pluginfile.php/15312/mod_resource/content/0/ocrm3c1n.pdf</a></p> <p>As with all applied mechanics topics the fundamental skills must be practised using as many different contexts as possible and a complete and fully labelled diagram is an essential ingredient for success.</p> <p>The past examination paper questions detailed below provide some examples of the type of contexts in which questions on this topic may be set and learners should gain experience through seeing as wide a range of problems as possible.</p> <p>As with all work in this area of applied mathematics, the importance of consolidation and practice cannot be emphasised enough and a significant proportion of the available time must be devoted to attempting problems of varying difficulty.</p> <p><b>Extension activity:</b> Supplementary notes and extension material is available at <a href="http://www.tes.com/teaching-resource/maths-mechanics-3-set-of-lesson-powerpoints-11000759">www.tes.com/teaching-resource/maths-mechanics-3-set-of-lesson-powerpoints-11000759</a></p> <p>Additional material can also be found at <a href="http://www.stem.org.uk/">www.stem.org.uk/</a> This provides additional notes and a range of suitable extension problems (E).</p>

**Past examination papers**

Past examination papers and mark schemes are available to download at <https://teachers.cie.org.uk>

Jun 14 Paper 21/22 Q4 Particle on a wire

Jun 14 Paper 23 Q3 Particle on a string

Jun 15 Paper 21/22 Q3 Particle on a sphere

Jun 15 Paper 23 Q10 EITHER Particle on a string

Nov 15 Paper 21 Q4 Particle on a string plus circular motion

Jun 16 Paper 21/22 Q4 Particle inside sphere

Jun 16 Paper 23 Q1 Particle on a string

## Equilibrium of a rigid body under coplanar forces

Syllabus ref	Learning objectives	Suggested teaching activities
3. Equilibrium of a rigid body under coplanar forces	<p>Understand and use the result that the effect of gravity on a rigid body is equivalent to a single force acting at the centre of mass of the body, and identify the centre of mass by considerations of symmetry in suitable cases;</p> <p>Calculate the moment of a force about a point in 2 dimensional situations only;</p> <p>Recall that if a rigid body is in equilibrium under the action of coplanar forces then the vector sum of forces is zero and the sum of the moments of the forces about any point is zero, and the converse of this;</p> <p>Use Newton's third law in situations involving the contact of rigid bodies in equilibrium;</p> <p>Solve problems involving the equilibrium of rigid bodies.</p>	<p><b>Prior knowledge:</b> Learners should be familiar with the idea of a rigid body, which cannot be modelled as a particle. They need to consider the moments of forces acting on them as well as the resultant force, and that for a rigid body to be in equilibrium the total moment at any point must be zero, as well as the resultant force on the body being zero.</p> <p><b>Main theme:</b> The topic essentially builds on the work already covered in the equilibrium of a single body and extends it to the use of Newton's third law with rigid bodies in contact with one another.</p> <p>Learners must be made aware that the key element in this topic is the use of a clear, fully labelled diagram in all cases. Once again the nature of the notation used is a vital step in achieving success in solving problems.</p> <p>Excellent basic notes are available at <a href="http://integralmaths.org/">http://integralmaths.org/</a> along with a range of examples and exercises: <a href="http://integralmaths.org/pluginfile.php/15324/mod_resource/content/0/ocrm3r1n.pdf">http://integralmaths.org/pluginfile.php/15324/mod_resource/content/0/ocrm3r1n.pdf</a>.</p> <p>Further examples and exercises are also available at <a href="http://www.mathsnetalevel.com/">www.mathsnetalevel.com/</a>.</p> <p>It is essential that learners are familiar with a range of approaches for calculating the vector sum of the moments for a system as well as appreciating the importance of producing a clear and straightforward solution (<b>I</b>).</p> <p>A useful video is available at <a href="http://www.examsolutions.net/maths-revision/mechanics/statics/moments/inclined-forces/tutorial-1.php">www.examsolutions.net/maths-revision/mechanics/statics/moments/inclined-forces/tutorial-1.php</a></p> <p>Problems involving hinges will be set and the difference in labelling should be known.</p> <p>There ought to be a clear understanding at this level that <math>F \leq \mu R</math> should be used for a body in equilibrium unless the limiting case is specified.</p> <p>The importance of correct labelling of diagrams is emphasised in problems where Newton's third law is applicable. Errors often occur as a direct result of incorrect notation.</p>

Syllabus ref	Learning objectives	Suggested teaching activities
		<p><b>Extension activity:</b> Supplementary notes and extension material is available at <a href="http://www.tes.com/teaching-resource/maths-mechanics-3-set-of-lesson-powerpoints-11000759">www.tes.com/teaching-resource/maths-mechanics-3-set-of-lesson-powerpoints-11000759</a></p>
<p><b>Past examination papers</b></p>		
<p>Past examination papers and mark schemes are available to download at <a href="https://teachers.cie.org.uk">https://teachers.cie.org.uk</a></p> <p>Jun 14 Paper 21/22 Q11 EITHER Wire frame on pegs  Jun 14 Paper 23 Q4 Rod resting on peg and plane  Jun 15 Paper 21/22 Q4 Hinged rod and elastic string  Jun 15 Paper 23 Q4 Hinged rod resting on cube  Nov 15 Paper 21 Q1 Ladder problem  Jun 16 Paper 21/22 Q11 EITHER Hinged rod and elastic string  Jun 16 Paper 23 Q2 Rod resting on peg</p>		

## Rotation of a rigid body

Syllabus ref	Learning objectives	Suggested teaching activities
4. Rotation of a rigid body	<p>Understand and use the definition of the moment of inertia of a system of particles about a fixed axis as <math>\sum mr^2</math> and the additive property for a rigid body composed of several parts;</p> <p>Use the parallel and perpendicular axes theorems;</p> <p>Recall and use the equation of angular motion <math>C = I\ddot{\theta}</math> for the motion of a rigid body about a fixed axis;</p> <p>Recall and use the formula <math>\frac{1}{2}I\omega^2</math> for the kinetic energy of a rigid body rotating about a fixed axis;</p> <p>Use conservation of energy in solving problems concerning mechanical systems where rotation of a rigid body about a fixed axis is involved.</p>	<p><b>Prior knowledge:</b> Learners must be confident with the ideas of work, energy momentum and impulse. They must understand the methods for finding the moment of a force, the work on circular motion and simple harmonic motion (SHM) with particular reference to the simple pendulum.</p> <p><b>Main theme:</b> The topic can be regarded as the equivalent of work on Newton's second law applied to a rigid body rather than a particle. Mass is replaced by moment of inertia, force is replaced by the moment of a force and linear acceleration is replaced by angular acceleration.</p> <p>As stated in the specification no integration is required in calculating the moments of inertia in this topic. The relevant formulae are found on the formula sheet but care must be exercised in identifying the components of a composite body and the precise axis used.</p> <p>Basic notes, examples and consolidation exercises can be found at <a href="http://integralmaths.org/">http://integralmaths.org/</a>.</p> <p>A variety of introduction examples in different approaches are available at <a href="http://www.youtube.com/user/ExamSolutions/videos">www.youtube.com/user/ExamSolutions/videos</a>.</p> <p>It is vital that learners appreciate that there are two main applications in this topic and it is essential that they have a thorough appreciation of which one to use. An attempt using the incorrect method may prove very costly.</p> <p>Since centres of mass are available through the symmetry of the system, this should be quite straightforward.</p> <p><b>Extension activity:</b> Supplementary notes (E) are available at <a href="http://www.tes.com/teaching-resource/maths-mechanics-3-set-of-lesson-powerpoints-11000759">www.tes.com/teaching-resource/maths-mechanics-3-set-of-lesson-powerpoints-11000759</a></p>

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Jun 14 Paper 21/22 Q5 Moment of Inertia plus small oscillations

Jun 14 Paper 23 Q5 Moment of Inertia plus energy

Jun 15 Paper 21/22 Q11 EITHER Moment of Inertia plus energy

Jun 15 Paper 23 Q5 Moment of Inertia plus small oscillations

Nov 15 Paper 21 Q10 EITHER Moment of Inertia plus small oscillations

Jun 16 Paper 21/22 Q5 Moment of Inertia plus small oscillations

Jun 16 Paper 23 Q4 Moment of Inertia plus energy

## Simple harmonic motion

Syllabus ref	Learning objectives	Suggested teaching activities
5. Simple harmonic motion	<p>Recall a definition of SHM and understand the concepts of period and amplitude;</p> <p>Use standard SHM formulae in the course of solving problems;</p> <p>Set up the differential equation of motion in problems leading to SHM, recall and use appropriate forms of solution and identify the period and amplitude of the motion;</p> <p>Recognise situations where an exact equation of motion may be approximated by an SHM equation...and appreciate the conditions necessary for such approximations to be useful.</p>	<p><b>Prior knowledge:</b> Learners must be familiar with Hooke’s Law, the work-energy principle and section 6 of the pure section of the paper 1 syllabus referring to the solution of second order differential equations. They should have met a variety of approaches in solving the resulting equations in this topic. Ideally, they will be able to draw appropriately on this knowledge and apply their skills.</p> <p>The emphasis in examination questions is clearly to challenge the learner’s ability to apply these standard formulae in a variety of differing, and less familiar, contexts.</p> <p><b>Main theme:</b> Basic notes, examples and exercises are available at <a href="http://integralmaths.org/http://integralmaths.org/pluginfile.php/15315/mod_resource/content/0/ocrm3o1n.pdf">http://integralmaths.org/http://integralmaths.org/pluginfile.php/15315/mod_resource/content/0/ocrm3o1n.pdf</a>.</p> <p>Alternative examples can be found at <a href="http://www.youtube.com/user/ExamSolutions/videos">www.youtube.com/user/ExamSolutions/videos</a>.</p> <p>Familiarity with work on elastic strings is a common requirement for SHM problems and it is important that the resulting equation of motion is expressed correctly as <math>\ddot{x} = -\omega^2 x</math> or a similar equivalent form. The use of <math>a</math> rather than <math>\ddot{x}</math> will not be accepted.</p> <p>Learners must be familiar with a variety of methods for solving or interpreting the resulting equations. The key to solutions in this topic lies often with the correct identification of the initial or starting conditions and this permits the most efficient solution to be chosen.</p> <p>As with all work in this area of applied mathematics, the importance of consolidation and practice cannot be emphasised enough and a significant proportion of the available time must be devoted to attempting problems of varying difficulty.</p> <p><b>Extension activity:</b> Further material (E) is available at <a href="http://www.tes.com/teaching-resource/maths-mechanics-3-set-of-lesson-powerpoints-11000759">www.tes.com/teaching-resource/maths-mechanics-3-set-of-lesson-powerpoints-11000759</a> and a number of more demanding problems can be found on the <a href="http://www.nrich.maths.org">www.nrich.maths.org</a> website.</p>

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Jun 14 Paper 21/22 Q2 SHM single particle

Jun 14 Paper 23 Q11 EITHER SHM elastic string

Jun 15 Paper 21/22 Q2 SHM single particle

Jun 15 Paper 23 Q3 SHM single particle

Nov 15 Paper 21 Q3 SHM elastic string

Jun 16 Paper 21/22 Q3 SHM single particle

Jun 16 Paper 23 Q11 EITHER SHM elastic string

## Further work on distributions

Syllabus ref	Learning objectives	Suggested teaching activities
6. Further work in distributions	Use the definition of the distribution function of $X$ to deduce the form of a distribution function of $Y$ in simple cases, e.g. to find the distribution function for $Y$ where $Y = X^3$ and $X$ has a given distribution;	<p><b>Prior knowledge:</b> Learners should be familiar with continuous probability density functions but the definition of a distribution function cannot be assumed at this stage.</p> <p><b>Main theme:</b> Notes on this topic appear in <a href="http://integralmaths.org/">http://integralmaths.org/</a>: <a href="http://integralmaths.org/pluginfile.php/16026/mod_resource/content/0/ocr3r1n.pdf">http://integralmaths.org/pluginfile.php/16026/mod_resource/content/0/ocr3r1n.pdf</a></p> <p>It is essential that learners show full detail when deducing the distribution function of the transformed random variable. It is perfectly possible to arrive at the correct answer even though crucial steps have been omitted and they must appreciate that this is not acceptable in this topic. It is also important that correct notation is used throughout. Random variables must appear in upper case while the distribution function or pdf should be expressed in lower case.</p>
6. Further work in distributions	<p>Understand conditions under which a geometric distribution or negative exponential distribution may be a suitable probability model;</p> <p>Recall and use the formula for the calculation of geometric or negative exponential probabilities;</p> <p>Recall and use the means and variances of a geometric distribution and negative exponential distribution.</p>	<p><b>Prior knowledge:</b> Learners should be familiar with the prior work on discrete and continuous random variables.</p> <p><b>Introduction:</b> An introductory video is available at <a href="http://www.examsolutions.net/maths-revision/statistics/discrete-random-variables/geometric-distribution/intro/tutorial-1.php">www.examsolutions.net/maths-revision/statistics/discrete-random-variables/geometric-distribution/intro/tutorial-1.php</a> for the basic geometric distribution. A similar introduction to the exponential distribution can be found at <a href="http://www.youtube.com/watch?v=IT-0oCOQrBY">www.youtube.com/watch?v=IT-0oCOQrBY</a>.</p> <p>The means and variances of both geometric and exponential distribution appear on the Formula sheet. It should be noted that, although the syllabus suggests that results need simply to be learned, or recalled, it is advisable that learners show essential details in their written solutions.</p> <p>As with all work in this area of applied mathematics, the importance of consolidation and practice cannot be emphasised enough and a significant proportion of the available time must be devoted to attempting problems of varying difficulty.</p>

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Jun 14 Paper 21/22 Q6 Geometric distribution Q7 Exponential distribution

Jun 14 Paper 23 Q7 Geometric distribution Q9 Transformation of random variables

Jun 15 Paper 21/22 Q9 Exponential distribution

Jun 15 Paper 23 Q9 Transformation of random variables

Nov 15 Paper 21 Q6 Geometric distribution Q7 Transformation of random variables

Jun 16 Paper 21/22 Q6 Geometric distribution Q8 Transformation of random variables

Jun 16 Paper 23 Q6 Geometric distribution Q7 Exponential distribution

## Inference using normal and t-distributions

Syllabus ref	Learning objectives	Suggested teaching activities
7. Inference using normal and t-distributions	<p>Apply a hypothesis test concerning the population mean for a small sample drawn from a normal population of unknown variance, using a <math>t</math>-test;</p> <p>Calculate a pooled estimate of a population variance from two samples;</p> <p>Formulate hypotheses concerning the difference of population means and apply as appropriate:</p> <ul style="list-style-type: none"> <li>- a 2-sample <math>t</math>-test,</li> <li>- a paired-sample <math>t</math>-test,</li> <li>- a test using a normal distribution;</li> </ul> <p>Determine a confidence interval for a population mean, based on a small sample from a normal population with unknown variance, using a <math>t</math>-distribution;</p> <p>Determine a confidence interval for a difference of population means, using a <math>t</math>-distribution, or a normal distribution, as appropriate.</p>	<p><b>Prior knowledge:</b> Learners should be familiar with the structure of a hypothesis test and with the formulae for unbiased estimates of both mean and variance from data given in a variety of forms.</p> <p><b>Introduction:</b> The link below relates to an introductory set of short videos covering much of the material in this section. Learners should be familiar with the different approaches to carrying out a hypothesis test although it is likely that they will focus on the process that best suits their style of working.  <a href="http://www.khanacademy.org/math/statistics-probability/significance-tests-one-sample/tests-about-population-mean/v/hypothesis-testing-and-p-values">www.khanacademy.org/math/statistics-probability/significance-tests-one-sample/tests-about-population-mean/v/hypothesis-testing-and-p-values</a></p> <p><b>Main theme:</b> Basic notes, examples and exercises are available at <a href="http://integralmaths.org/">http://integralmaths.org/</a> where these topics feature in the S3 specification.  <a href="http://integralmaths.org/pluginfile.php/16040/mod_resource/content/0/ocr3d1n.pdf">http://integralmaths.org/pluginfile.php/16040/mod_resource/content/0/ocr3d1n.pdf</a></p> <p>Learners must gain experience in selecting the appropriate test for data supplied in a variety of forms as well as appreciating the conditions necessary for a particular test to be valid.</p> <p>The links below are also available as an introduction to the principles of hypothesis testing and can be used as a starting point for a discussion of the concept.</p> <p><a href="http://www.khanacademy.org/math/statistics-probability/significance-tests-one-sample/tests-about-population-mean/v/one-tailed-and-two-tailed-tests">www.khanacademy.org/math/statistics-probability/significance-tests-one-sample/tests-about-population-mean/v/one-tailed-and-two-tailed-tests</a>  <a href="http://www.khanacademy.org/math/statistics-probability/significance-tests-one-sample/tests-about-population-mean/v/z-statistics-vs-t-statistics">www.khanacademy.org/math/statistics-probability/significance-tests-one-sample/tests-about-population-mean/v/z-statistics-vs-t-statistics</a>  <a href="http://www.khanacademy.org/math/statistics-probability/significance-tests-one-sample/tests-about-population-mean/v/small-sample-hypothesis-tests">www.khanacademy.org/math/statistics-probability/significance-tests-one-sample/tests-about-population-mean/v/small-sample-hypothesis-tests</a></p> <p>It is important to emphasise the situations in which <math>\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}</math> is appropriate rather than</p>

Syllabus ref	Learning objectives	Suggested teaching activities
		the alternative $\frac{(n_A - 1)s_A^2 + (n_B - 1)s_B^2}{n_A + n_B - 2}$ The notes in <a href="http://integralmaths.org/">http://integralmaths.org/</a> provide a detailed explanation.
<b>Past examination papers</b>		
<p>Past examination papers and mark schemes are available to download at <a href="https://teachers.cie.org.uk">https://teachers.cie.org.uk</a></p> <p>Jun 14 Paper 21/22 Q8 2-sample normal test</p> <p>Jun 14 Paper 23 Q6 paired-sample t-test Q10 small sample t-test plus confidence interval</p> <p>Jun 15 Paper 21/22 Q6 pooled samples Q7 small sample confidence interval Q10 paired-sample t-test</p> <p>Jun 15 Paper 23 Q8 small-sample confidence interval Q10 OR 2 sample normal test plus confidence interval</p> <p>Nov 15 Paper 21 Q5 small-sample confidence interval Q10 OR 2-sample t-test and small sample t-test</p> <p>Jun 16 Paper 21/22 Q7 small sample t-test Q11 OR small sample confidence interval plus 2 sample t-test</p> <p>Jun 16 Paper 23 Q8 paired-sample t-test Q11 OR 2-sample normal test plus confidence interval</p>		

$\chi^2$  - tests

Syllabus ref	Learning objectives	Suggested teaching activities
8. $\chi^2$ -tests	<p>Fit a theoretical distribution, as prescribed by a given hypothesis, to given data;</p> <p>Use a <math>\chi^2</math> -test, with the appropriate number of degrees of freedom, to carry out the corresponding goodness of fit analysis;</p> <p>Use a <math>\chi^2</math> -test, with the appropriate number of degrees of freedom, for independence in a contingency table. Classes need to be combined to ensure that the expected frequency is at least five.</p>	<p><b>Prior knowledge:</b> Learners should be familiar with discrete uniform, binomial, Poisson, geometric, normal and other continuous random variables defined by their pdf or distribution function.</p> <p><b>Introduction:</b> The basic principles can be found at <a href="http://www.khanacademy.org/math/statistics-probability/inference-categorical-data-chi-square-tests/chi-square-goodness-of-fit-tests/v/pearson-s-chi-square-test-goodness-of-fit">www.khanacademy.org/math/statistics-probability/inference-categorical-data-chi-square-tests/chi-square-goodness-of-fit-tests/v/pearson-s-chi-square-test-goodness-of-fit</a></p> <p><b>Main theme:</b> Detailed notes, examples and exercises are available at <a href="http://integralmaths.org/">http://integralmaths.org/</a> where this topic falls in the S3 section of the Statistics specification. <a href="http://integralmaths.org/pluginfile.php/16046/mod_resource/content/0/ocr3g1n.pdf">http://integralmaths.org/pluginfile.php/16046/mod_resource/content/0/ocr3g1n.pdf</a> These cover goodness-of-fit tests with a good range of examples. An example of a hypothesis is: <math>H_0</math> : model fits the data <math>H_1</math> : model does not fit the data</p> <p>It is important to set out the working in tabular form and the rules for calculating the number of degrees of freedom must be fully understood.</p> <p>As the learning objectives indicate the minimum number in an expected frequency is five; this applies to both goodness of fit and contingency tables. If rows or columns have to be combined, the context given will be straightforward to apply.</p> <p>Further notes for dealing with contingency tables are available at <a href="http://integralmaths.org/pluginfile.php/16047/mod_resource/content/1/ocr3g2n.pdf">http://integralmaths.org/pluginfile.php/16047/mod_resource/content/1/ocr3g2n.pdf</a></p> <p>An example of a suitable hypothesis is <math>H_0</math> : factors are not associated <math>H_1</math> : factors are associated (in context)</p>

		<b>Extension material:</b> Learners might be encouraged to investigate the nature of the chi-squared distribution and its links to other distributions.
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### Past examination papers

Past examination papers and mark schemes are available to download at <https://teachers.cie.org.uk>

Jun 14 Paper 21/22 Q9 Goodness of fit test

Jun 14 Paper 23 Q8 Contingency table

Jun 15 Paper 21/22 Q11 OR Goodness of fit test

Jun 15 Paper 23 Q6 Contingency table

Nov 15 Paper 21 Q8 Goodness of fit test

Jun 16 Paper 21/22 Q9 Goodness of fit test

Jun 16 Paper 23 Q9 Contingency table

## Bivariate data

Syllabus ref	Learning objectives	Suggested teaching activities
9. Bivariate data	<p>Understand the concept of least squares, regression lines and correlation in the context of a scatter diagram;</p> <p>Calculate...the equations of regression lines and the product moment correlation coefficient;</p> <p>Appreciate the distinction between the regression line of <math>y</math> on <math>x</math> and that of <math>x</math> on <math>y</math>;</p> <p>Recall and use the facts that both regression lines pass through <math>(\bar{x}, \bar{y})</math> and that <math>r^2 = b_1 b_2</math>;</p> <p>Select and use, in the context of the problem, the appropriate regression line to estimate a value, and understand the uncertainties associated with such estimations.</p> <p>Relate, in simple terms, the value of the product moment correlation coefficient to the appearance of the scatter diagram with particular reference to the interpretation of cases where the value of the product moment correlation coefficient is close to +1, -1 or 0;</p> <p>Carry out a hypothesis test based on the product moment correlation coefficient.</p>	<p><b>Introduction:</b> The following links provide two alternative introductions to the topic:  <a href="http://www.khanacademy.org/math/statistics-probability/describing-relationships-quantitative-data#scatterplots-and-correlation">www.khanacademy.org/math/statistics-probability/describing-relationships-quantitative-data#scatterplots-and-correlation</a>  or alternatively: <a href="http://www.examsolutions.net/maths-revision/statistics/correlation-regression/correlation/introduction.php">www.examsolutions.net/maths-revision/statistics/correlation-regression/correlation/introduction.php</a></p> <p><b>Main theme:</b> Detailed notes can be found at <a href="http://integralmaths.org/">http://integralmaths.org/</a> along with examples and further exercises from the Statistics S1 module.  <a href="http://integralmaths.org/pluginfile.php/6876/mod_resource/content/0/ocrslc1n.pdf">http://integralmaths.org/pluginfile.php/6876/mod_resource/content/0/ocrslc1n.pdf</a>  <a href="http://integralmaths.org/pluginfile.php/6879/mod_resource/content/0/ocrslc2n.pdf">http://integralmaths.org/pluginfile.php/6879/mod_resource/content/0/ocrslc2n.pdf</a></p> <p>Data for problems on this topic may be supplied in raw form or as summative data.</p> <p>Uncertainties such as whether the given value falls within the range of the data, the value of the product moment correlation coefficient (PMCC) which may have been tested for significance and which regression line is used. The following is a possible introduction:  <a href="http://www.khanacademy.org/math/statistics-probability/describing-relationships-quantitative-data/residuals-least-squares-rsquared/v/regression-line-example">www.khanacademy.org/math/statistics-probability/describing-relationships-quantitative-data/residuals-least-squares-rsquared/v/regression-line-example</a></p> <p>It should be noted that this aspect will only be tested with very straightforward examples. The standard form for the hypotheses is <math>H_0 : \rho = 0</math> <math>H_1 : \rho \neq 0</math> for example.</p> <p><b>Extension material:</b> Learners can be encouraged to investigate the derivation of formulae used and the distributional assumptions associated with this topic.</p>

**Past examination papers**

Past examination papers and mark schemes are available to download at <https://teachers.cie.org.uk>

Jun 14 Paper 21/22 Q10 PMCC/Test/Regression

Jun 14 Paper 23 Q11 OR Regression plus PMCC test

Jun 15 Paper 21/22 Q8 PMCC/Test

Jun 15 Paper 23 Q7 Regression plus PMCC test

Nov 15 Paper 21 Q9 Regression/estimation/PMCC test

Jun 16 Paper 21/22 Q10 Regression and estimation

Jun 16 Paper 23 Q10 PMCC/Test/Regression/estimation

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